Practice 3

Topic Linearization of nonlinear processes

All processes (Control Objects – CO) in the nature are the nonlinear. Solving problems of the analysis and synthesis of ACS is rather difficult for nonlinear CO. Therefore if it is possible, nonlinear processes we will linearize, and then we use classical mathematical apparatus of linear ACS for the solution of various tasks.

So, let nonlinear process is described by the following equation

$$x' = f(x, u), \tag{1}$$

where $x(n*1)$ is a vector of a state;

$u\ (n*1)$ is a measured vector of control;

$f\ (n*1)$ is a vector function.

Nonlinear process we will linearize rather some working condition if on an input we give a small indignation δu.

If on an input of process we give δu, the vectors x and f will receive also the increment, i.e.

$$x' + \delta x' = f(x+\delta x,\ u+\delta u). \tag{2}$$

If we subtract equation (1) from the equation (2), we will receive

$$\delta x' = f(x+\delta x,\ u+\delta u) - f(x,u) = \left[\frac{\partial f}{\partial x}\right]_{x_o,u_o}\delta x + \left[\frac{\partial f}{\partial x}\right]_{x_o,u_o}\delta u, \tag{3}$$

where $[\ldots]_{x_o,u_o}$ is the private derivative in the vicinity of *a working point* means (x_o, u_o).

We will designate

$$A = \begin{vmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \ldots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \ldots & \frac{\partial f_2}{\partial x_n} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \ldots & \frac{\partial f_n}{\partial x_n} \end{vmatrix};\quad B = \begin{vmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \ldots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{u x_2} & \ldots & \frac{\partial f_2}{\partial u_m} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \ldots & \frac{\partial f_n}{\partial u_m} \end{vmatrix}.$$

Then the equation (3) will correspond in the following look:

$$\delta \dot{x} = A\,\delta x + B\,\delta u,$$

which is called linearized at small indignations.

Example. Linearize nonlinear process which mathematical description in state space is set in the following look:

$$\begin{cases} \dot{x}_1 = -x_1^2 + 2x_1 u \\ \dot{x}_2 = x_1 x_2 - u^3 \end{cases}.$$

Write down the received mathematical description of the linearized system in matrix and scalar forms.

Algorithm and solution

1) We define dimensions of vectors; we write down in a general view of a matrix *A* and *B*.

$$\begin{cases} f_1(x,u) = -x_1^2 + 2x_1 u \\ f_2(x,u) = x_1 x_2 - u^3 \end{cases}; \qquad n=2;\ m=1;\ m<n.$$

$$A = \begin{vmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{vmatrix}; \quad B = \begin{vmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{vmatrix}.$$

2) We define matrixes *A* and *B* in that specific case for an example.

$$A = \begin{vmatrix} (-2x_{1,0} + 2u_0) & 0 \\ x_{2,0} & x_{1,0} \end{vmatrix}; \; B = \begin{vmatrix} 2x_{1,0} \\ -3u_0^2 \end{vmatrix}.$$

3) We write down the mathematical description of the linearized system in matrix and scalar forms.

Structure of the equation of the linearized system:

$$\delta\dot{x} = A\,\delta x + B\,\delta u.$$

$$\delta\dot{x} = \begin{vmatrix} (-2x_{1,0} + 2u_0) & 0 \\ x_{2,0} & x_{1,0} \end{vmatrix} \delta x + \begin{vmatrix} 2x_{1,0} \\ -3u_0^2 \end{vmatrix} \delta u\,; \qquad (4)$$

$$\begin{cases} \delta\dot{x}_1 = (-2x_{1,0} + 2u_0)\delta x_1 + 2x_{1,0}\delta u \\ \delta\dot{x}_2 = x_{2,0}\delta x_1 + x_{1,0}\delta x_2 - 3u_0^2\delta u \end{cases}. \qquad (5)$$

Conclusion: We have received the mathematical description of the linearized system in the form of the equations (4) and (5).

Task (on variant):

Linearize nonlinear process which mathematical description in state space is set in the following look: (see variant). Write down the received mathematical description of the linearized system in matrix and scalar forms.

1)
$$\dot{X}_1 = -3X_1^2+5X_2+3X_2U_2^2$$
$$\dot{X}_2=2X_1X_2+3X_2^2-X_1U_1^2$$

2)
$$\dot{X}_1=3X_1^2X_2-5X_2+U_1$$
$$\dot{X}_2=2X_1+3X_2^2-3X_1U_2^2$$

3)
$$\dot{X}_1=-X_1^2-4X_1X_2^2+X_2U_1^2$$
$$\dot{X}_2=X_1X_2+3X_2^2+2X_2U_2$$

4)
$$\dot{X}_1=-X_1^2X_2+7X_2^2-3U_1$$
$$\dot{X}_2=-X_1^2X_2^2-4X_1X_2+U_2^2$$

5)
$$\dot{X}_1=-X_1^2X_2+7X_2^2-3U_1$$
$$\dot{X}_2=-X_1^2X_2^2-4X_1X_2+U_2^2$$

6)
$$\dot{X}_1=-7X_1^2+3X_1X_2^2-2U_1^2$$
$$\dot{X}_2=-4X_1X_2+5X_2^2+3X_1X_2U_2$$

7)
$$\dot{X}_1=X_1X_2^2-3X_2+2X_1U_1^2$$
$$\dot{X}_2=-X_1-5X_2^2-U_2^2$$

8) $$\dot{X}_1 = -X_1^2 - 3X_1X_2^2 + U_1$$
$$\dot{X}_2 = -X_1 + 5X_2^2 - 3U_2$$

9) $$\dot{X}_1 = -X_1X_2 + 5X_2^2 + 2U_1^2$$
$$\dot{X}_2 = X_1^2 + X_1X_2 - 2U_2^2$$

10) $$\dot{X}_1 = -X_1^2X_2 - 5X_2 + 2X_1U_1$$
$$\dot{X}_2 = 2X_1^2 + 4X_2^2 - 3X_1U_2^2$$

11) $$\dot{X}_1 = -2X_1X_2^2 - 5X_2^2 + 3X_2U_1^2$$
$$\dot{X}_2 = -2X_1 + 3X_2^2 - X_1U_2^2$$

12) $$\dot{X}_1 = 3X_1^2X_2 - 5X_2 + U_1$$
$$\dot{X}_2 = 2X_1 + 3X_2^2 - 3X_1U_2^2$$